Placing an Emphasis on Mathematical Language as a Key to Unlocking Mathematical Understanding

Dr. Lorna Quinnell

Charles Darwin University

Phone 0403 735 168

Email lorna.quinnell@cdu.edu.au

Address P O Box 13154, George Street Post Shop, Brisbane, 4003

Key Words: Understanding, Mathematics, Language Strategies

Abstract: *Understanding is one of the four proficiencies or key ideas in the Australian Mathematics Curriculum and arguably the pivot on which mathematics depends. With reference to a selection of mathematical language, this review discusses language strategies that can be used in mathematics classrooms to enhance students' understanding of mathematical concepts. The activities were carried out with preservice teachers with the aim of revising and scaffolding their knowledge of mathematical language and introducing them to language strategies that can be used in school classrooms.*

The paper argues that enhancing understanding through a focus on language is crucial to aiding students to build a solid foundation of mathematical ideas.

Introduction

Understanding is emphasised as one of four proficiencies or key ideas in the Australian Mathematics Curriculum (Australian Curriculum, Assessment and Reporting Authority, 2018). This applies to all levels including middle school levels. One element of understanding in mathematics is understanding of the language of the subject, acknowledged as a key component of reading, understanding, and learning of mathematics (see Dunston & Tyminski, 2013; Martiniello, 2008). It is important for instance, that middle school teachers foster students' ability to understand mathematical language, as a way of scaffolding their learning about the subject and their understanding of the requirements of assessment questions (e.g,, NAPLAN questions).

Scaffolding students' understanding of mathematical language relies on teachers' acknowledgement of the difficulties posed by key language, followed by a focus on this area of mathematics. Teachers' require deep understanding of the content of a subject in addition to an ability to convey the content effectively to the students (Australian Institute for Teaching and School Leadership Limited, 2018). Thus, aiding students to develop a deep understanding of mathematical vocabulary and language relies on teachers' understanding of the language and knowledge of strategies that can be used to enhance the learning and understanding of language.

Many strategies have been proposed by educators for scaffolding the learning of mathematical language. Much of the related work was carried out prior to the past decade (e.g., Friedland, McMillen, & del Prado Hill, 2011). The strategies often had their origins in language teaching, with details published in peer-reviewed journals for use in mathematics teaching. They frequently made use of visuals, some making links between different semiotic systems (e.g., Dunston & Tyminski, 2013). These complement the range of vocabulary introduction ideas that are categorised and discussed in an informative article by Rubenstein (2007), designed to address various challenges posed by mathematical language.

Although many of the strategies discussed below were originally aimed at middle school levels (e.g., Dunston & Tyminski, 2013; Gay & White, 2002; Rubenstein, 2007), with minor modifications such as careful choice of subject words and categories, they are likely to be more widely applicable. The activities described in this article were carried out with preservice teachers with the aim of revising and scaffolding their knowledge of mathematical language and introducing them to language strategies that can be used in school classrooms.

Strategies for introducing mathematical terminology

Formulation of definitions has been recommended by educators for the introduction of mathematical vocabulary in many school levels (e.g., Boulet, 2007; Pierce & Fontaine, 2009; Shield, 2004). However, this strategy presents benefits and difficulties. Composing descriptions or definitions can be used to facilitate language use, advance understanding and reasoning, expose imperfect constructions, and stimulate verbal discourse and thinking about mathematical ideas. The benefits of talking about mathematical concepts is evident in Boulet (2007), who provided examples of the use of dialogue as a key to encouraging active learning, communication, and thought.

Formulation of definitions can follow formal or less formal approaches. A formal approach in which objects are defined according to the item, class, and properties can be used (e.g., Shield, 2004). For instance, a *right-angled triangle* (item) is a triangle (class) which is a closed, plane shape with three straight sides, in which one angle is a right angle (properties).

Comprehensive definitions are an example of the precision, brevity and density of mathematical language. However, formulating formal definitions can present difficulties to learners, as is evident in Boulet's (2007) discussion of the difficulties that teachers experienced whilst collaboratively creating a definition of a *polygon* (pp. 1-2). Definitions often do not give enough information about the complexities of the meaning of a word (Ewing Monroe & Orme, 2002), and in the process of defining some concepts the richness of a concept is lost (Leung, 2005). Comprehensive definitions of many mathematical concepts are complicated and may rely on learners' prior knowledge, or be above the level of the learner, or use other unfamiliar terms (Leung, 2005; Shield, 2004). The definition for a *regular hexagon*, for instance, relies on a reference to many ideas, including a closed planar shape, with six straight sides, and equal sides and angles. Mathematical definitions are not necessarily unique (Boulet, 2007; Shield, 2004), for instance two very different definitions may be possible for the same shape. Many mathematical concepts are impossible to define concisely and unambiguously, even relatively simple terms such as *one dimension* or *square* (Leung, 2005, pp. 128-130). As stated by a student, "there's no such thing as a one dimensional shape coz a line is kind of like a rectangle filled in" (Leung, 2005, pp. 128-129). Often words can be used and understood without use of concise and unambiguous definitions (Leung, 2005).

Importantly, composing definitions depends on deep understanding of concepts, an understanding that needs to be developed before definitions can be considered. The difficulties of composing comprehensive mathematical definitions suggest that alternative strategies need to be considered. In line with the focus on understanding in the curriculum, such methods require a primary focus on the development of conceptual understanding, rather than on procedural knowledge. As an example, it is crucial that students develop deep understanding of the attributes area and perimeter before considering definitions or using formulae to calculate them. Without a focus in understanding, definitions may be meaningless and formulae may be misused. Fostering conceptual understanding can be achieved through use of manipulatives and diagrams, and reference to dictionaries and use of prefixes. Use of concrete manipulatives, for instance, aids students' understanding by allowing them to visualise and describe something tangible rather than describing otherwise abstract concepts. Involvement in written activities and dialogue about attributes such as area and perimeter, which may include informal approaches to vocabulary introduction are also advocated by educators (e.g., Pierce & Fontaine, 2009). The final step in the process is the development of formal definitions.

Informal definitions, which are gradually perfected by providing students opportunities to focus on examples, are useful in mathematics, encouraging learners to utilise and apply mathematical vocabulary. Knowledge construction through the process can be complemented by the inclusion of carefully labelled diagrams, and further questions, which encourage a deeper understanding by encouraging learners to think beyond the meaning of separate words. This corresponds to the idea of utilising user-friendly definitions and of including activities that make use of mathematical discourse to gradually enhance conceptual understanding (e.g., Dunston & Tyminski, 2013; Renne, 2004). In Renne's (2004) study, the described activities reinforced understanding of concepts and gradually honed in on increasingly well-defined concepts. Similarly, the set steps proposed by Pierce and Fontaine (2009) for primary levels can be used for vocabulary introduction. The steps comprise the use of user-friendly definitions, followed by discussion of the different meanings of a word and finally activities that promote deep processing of the new terminology. “Language acquisition takes time and occurs from connecting words to experiences” (Burns, 2007, p. 374); students’ understanding of a word, including multiple meanings of a word, evolves in parallel with the understanding of the concept.

The process of constructing gradually more perfect definitions can be enhanced through following the steps: formulating of individual descriptions, rethinking and modifications after collaboration with peers and use of dictionaries, following think-pair-share ideas (e.g., Chamberlin, 2009). The process of communicating with others whilst formulating definitions can be used to highlight the necessary precision and reproducibility required when talking about mathematical concepts. For instance, development of comprehensive descriptions of acute-angled, right-angled and obtuse-angled triangles reveals that acute-angled triangles have three acute angles, whereas right-angled and obtuse-angled triangles have one right angle and one obtuse angle respectively. Formulating a description of *regular polygons* exposes the importance of stating that both sides and angles are equal. A star with ten equal sides for instance, is not a regular shape because the angles are not equal. An understanding of the term *regular polygon* can be reinforced for regular hexagons for example, through a discussion of hexagons, equiangular hexagons, equilateral hexagons, and regular hexagons, including examples and non-examples of regular hexagons (Rubenstein, 2007).

The process of formulating user-friendly definitions and descriptions means that a fuller picture of students’ misconceptions are exposed in their descriptions, diagrams, and examples, meaning that it is possible to address the misconceptions. For example, two preservice teachers’ descriptions of the attributes radius/diameter/circumference/area in Table 1 below, draws attention to Silvia’s difficulties. They provide a good contrast to Cynthia’s more comprehensive descriptions, which show a deeper level of understanding, (except for her description of area, in which she appears to confuse the words diameter and circumference or perimeter). The descriptions offer an entry point to conversations about these attributes. Notably, describing an attribute in terms of the formula used to calculate it does not enhance or show understanding. An example can be seen in Silvia’s description of area, a description that does not show her understanding of the concept and applies only to rectangles.

Table 1

Two Preservice Teachers' Descriptions of Radius, Diameter, and Circumference

Student	Radius	Diameter	Circumference	Area
Silvia	Distance of a segment	She depicted a diameter on a diagram of a circle but did not clearly show that it passed through the centre	All the way around a circle	Area = 2d usually length multiplied by height $A = l \times h$ (It later became clear that 2d meant two dimensional)
Cynthia	Distance between centre of a circle and any point on its circumference	Diametre [sic] distance of a straight line from one side of a circle to another that passes through the centre ($2 \times r$)	The length of the outside of a circle	Space within the diametre [sic] of a shape

Descriptions and definitions can be used in both inductive and deductive teaching (Brahier, 2009). In an example, Brahier (2009) demonstrated how students can be asked to establish definitions for a diverse range of polygons. This is an example of inductive thinking, in which investigation of individual cases leads to generalisations and a definitions. Inductive thinking can be compared to deductive thinking, where generalisation precedes investigation of individual cases. In an example, students are provided with definitions of various polygons upfront and asked to use the definitions to identify a selection of polygons in a collection of shapes. Inductive thinking links to the idea of constructivist learning in which the learners are actively involved in the learning process, encouraging them to do the thinking, thus fostering understanding. Because they have created the definition or generalisation, it is more likely that it will be remembered (Brahier, 2009). However, although inductive activities offer advantages compared to deductive activities, they may be more difficult for teachers to design; relying on a teachers' deep understanding and creativity.

Some educators have advocated the introduction of vocabulary through multiple, meaningful learning experiences and varying contexts, followed by focused teaching and use of definitions (Ewing Monroe & Orme, 2002; Shield, 2004). This follows the notion that new vocabulary is only useful once a concept is understood, indicating the logic of introducing concepts before definitions (Burns, 2007). For example, as a foundation for understanding, children may be encouraged to explore and describe different trapeziums before the word *parallel* is introduced. In another example, the difference between volume and capacity may be illustrated effectively with manipulatives or visuals, before defining these concepts. A key idea in many vocabulary learning strategies is that learners need to use and explore the multiple meanings of mathematical vocabulary in order to become familiar with their use, since “knowing a word means knowing more than its core meaning” (Leung, 2005, p. 130).

Recommended too for mathematics teaching in middle school levels (but also arguably appropriate for other levels), are discussions of prefixes and, partly because of the number of mathematics words with Latin and Greek origins, roots and origins of words including reference to English words with the same roots (Rubenstein, 2007). Important prefixes include *peri-* and *circ-* meaning around, *tetra-* and *quadr-* meaning four, *equi-* and *equa-* meaning equal, *iso-* meaning equal, and *poly-* meaning many (e.g., Anderson et al., 2008). Knowledge of prefixes has the potential to aid understanding of words such as *polygon, hexagon,* and *isosceles triangle*. A teacher may draw attention to the meaning of *poly-* meaning many when introducing multi-sided 2D shapes. Other prefixes such as *hex-* meaning six can be used to unlock the meaning of *hexagon,* which is a 2D shape with six sides. Further understanding of polygons and the many important prefixes used in mathematics can be achieved by reference to concept maps such as those in Anderson et al. (2008). The concept maps for instance illustrate the pattern of prefixes used in the naming of shapes.

As a way of extending understanding, discussion may be used to form links between words that share the same prefix, for instance mathematical words and everyday words or technical English words from other disciplines. When discussing the word *circumference*, reference to the meaning of *circum-* (around) found in the word *circumnavigate* may be useful (Rubenstein, 2007). The prefix *trans-* from Latin, meaning through or across (Anderson et al., 2008), is found in words such as *translation, transatlantic, transit,* and *transparent* in English and Social Sciences. Through studying these words and their meanings in one (or more) other context, the mathematical words of translation, translate, transformation, transform and transversal can then be studied and their meanings explored.

Often two prefixes relate to the same concept, one from Greek origins and one from Latin. In an example, teachers need to recognise that both prefixes *tetra-* from Greek and *quad-* from Latin mean four, and are used in mathematical words such as *tetrahedron* (a 3D shape with four faces) and *quadrilateral* (a 2D shape with four sides) respectively. Prefixes that represent *one*, include *mono-* from Greek and *uni-* from Latin (Anderson et al., 2008). They are found in the words unit and union in mathematics and monologue, monorail, monotone, unique, unit, unicycle, universe, uniform, unify, union, in other curriculum areas or in everyday language. Discussion may be used to form links between the meanings of these words, based on the fact that each one includes a prefix meaning one. Notably, at times the same word is used in multiple contexts with different meanings. The meanings of words *unit* and *union* in mathematics are different from their meanings in everyday language although they all relate to the meaning of the prefix *uni-* meaning one.

Some vocabulary instruction strategies, previously used by reading teachers, have been recommended in mathematics for middle and senior level students. One example is the four-square model diagram (e.g., Figure 1), a useful means of organising a combination of word descriptions, diagrams, symbolic representations, and examples. The four-square model (verbal and visual word association diagram) consists of a rectangle divided into four sections, the first giving the word, the second a definition, the third a diagram, and the fourth, an association (Dunston & Tyminski, 2013; Gay & White, 2002). In Figure 1, the preservice teacher who constructed the diagram demonstrated a broad understanding of the word *hypotenuse*, and the ability to articulate the concept in multiple ways. She illustrated the word on differently orientated triangles and extended her diagram to include multiple associations such as linking it with the Theorem of Pythagoras. In this way, construction of graphic organisers, which include definitions and other representations, have the potential to enhance knowledge construction (e.g., Dunston & Tyminski, 2013). Four-square model diagrams can be used to depict a broad range of vocabulary from all strands, examples being *multiple, percentage, hypotenuse, perpendicular, kite, regular polygon, tessellation, square-based pyramid,* and *bar graph*. Once teachers are familiar with such diagrams, they can model their use for their students.

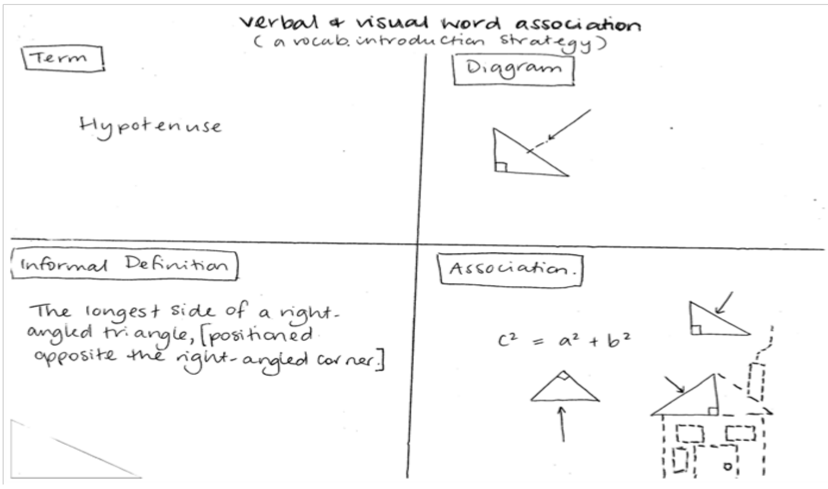

Figure 1. Example of a four-square model diagram depicting the word hypotenuse.

The *association* category in four-square model diagrams at times presents difficulties for students. For kites and square-based pyramids, associations may be tessellating patterns of kites and Egyptian pyramids respectively. Use of the four-square model, including the association category, is a way of aiding learners to construct understanding of terminology, guiding them towards construction of clear descriptions, and aiding them to link ideas with prior knowledge and understanding of multiple applications. The diagrams can also help to expose gaps in understanding (Gay & White, 2002). The four-square model is perhaps particularly useful in terms of enhancing understanding of potentially confusing words and terms in mathematics, such as words used exclusively or differently in mathematics classrooms compared to their everyday use (e.g., hypotenuse, mean). Importantly, the four-square model builds understanding in a visual way (Dunston & Tyminski, 2013) and forms links between different semiotic systems such as words, symbolism, and visual images (see O'Halloran, 2005).

Other graphic organisers are more complex but bear some similarities to four-square model diagrams. They incorporate definitions, examples and non-examples, attributes or characteristics (Ewing Monroe & Orme, 2002), varied representations, and reference to varied notation (Gough, 2007). Construction of four-square model diagrams and other graphic organisers (e.g., Dunston & Tyminski, 2013; Ewing Monroe & Orme, 2002) offer many benefits to students. They present "effective ways to help students assimilate the unique concepts and terms that they will encounter in mathematics" (Dunston & Tyminski, 2013, p. 44). Construction of such graphic organisers helps learners to structure information with reference to prior knowledge and encourages them to think about relationships (Dunston & Tyminski, 2013; Ewing Monroe & Orme, 2002). As do four-square model diagrams, construction of other graphic organisers aid understanding of the meaning of concepts, and may aid teachers to identify student misconceptions (Gay & White, 2002). Graphic organisers can be adapted for students at different levels, with careful thought given to the subject word and categories.

Other vocabulary introduction strategies have been advocated to aid students' understanding of mathematical concepts. They include concept maps, concept circles, word walls, and semantic feature analysis. Concept maps have been widely advocated for use in the space strand offering a means of depicting a diverse range of space concepts in an organised way (e.g., Anderson et al., 2008; Shield, 2004). Concept maps can be used in combination with written definitions to describe concepts (Shield, 2004).

Mathematical reference books and dictionaries are valuable tools when combined with other strategies to enhance conceptual understanding of mathematics vocabulary. For instance they can be used to improve on four-square model diagrams (e.g., Figure 1). Examples of mathematical reference books and dictionaries include handbooks such as the comprehensive *The Origo Handbook* (Anderson et al., 2008), or the *Primary maths handbook* (O'Brien and Purcell, 2013), and the online mathematics dictionary for children (Eather, 2011). Written by mathematics educators not mathematicians, *The Origo Handbook* is suitable for teachers, presenting mathematics terminology in accessible ways without the use of unnecessarily complex explanations. It includes diagrams, examples, and lists of mathematical symbols, abbreviations, formulae, and prefixes. The *Primary maths handbook* and the online Eather dictionary are attractive, colourful, and easy to read and navigate, hence are useful for primary and middle schooling. They contain definitions, and aid understanding through extensive use of illustrations, colour, examples, and exercises. The inclusion of diagrams and examples builds links between different ideas and representations, thereby providing more effective scaffolding of understanding than formal definitions. The fact that Eather (2011) is online makes it readily accessible to those who have access to the internet. Mathematical dictionaries are a better option than searching for mathematics words on the internet, since words are often used very differently in mathematics than in everyday language (Pierce & Fontaine, 2009). Important for Australian classrooms, Eather (2011), O'Brien and Purcell

(2013), and Anderson et al. (2008) are Australian, following Australian use of mathematics terminology and spelling. Since understanding of language evolves, becoming more precise with use (Kotsopoulos, 2007), and since academic definitions can be unhelpful (Leung, 2005), Eather (2011) presents an option for introducing students to mathematical language, bearing in mind that such a resource needs to be viewed with a critical eye. This can be followed by use of more comprehensive references such as *The Origo Handbook* (Anderson et al., 2008).

A combination of dialogue and use of four-square model diagrams and dictionaries can be used to scaffold understanding, as evident in the conversations followed by further activities about the concept *prime number* between a facilitator and preservice teacher below (see Quinnell, 2016).

> Student: I can't think what a prime number is. Um, 1, 3, 5, 7, 9 ...? Odd numbers?
>
> Facilitator: Is nine a prime?
>
> Student: No, it can be divided by three.
>
> Facilitator Yep, so what's a prime?
>
> Student: Numbers that can only be divided by themselves.
>
> Facilitator: And one. Can be divided by one and themselves. So write down examples of primes.
>
> Student: 1, 2, 3, 5, 7, ... [written]
>
> Facilitator: What do you see with the even numbers, how many are there?
>
> Student: Only one, is there only one? Is there only one altogether?
>
> Facilitator: Yes every other even number is divided [sic] by two as well as one and themselves. [The word divisible should have been used.] Is nine a prime?
>
> Student: No
>
> Facilitator: Eleven? … OK, put dot, dot, dot [an indication that the list is infinite].

The student was then asked to give an informal definition for prime numbers:

> Student: Numbers that can be divided by one and themselves [written].

As in the conversations in Zazkis, Liljedahl, and Sinclair (2009), through the communication process the student gradually constructed an expanded understanding of *prime number*, the dialogue aiding the student to formulate ideas, which were then internalised as internal thinking. The facilitator missed some opportunities to aid the student to expand her understanding and overlooked the student's inclusion of one in her list of primes. As advocated in Renne (2004), decisions need to be taken about what difficulties to address and when. The student's inclusion of 1 as a prime number would later be addressed when she referred to a mathematics dictionary. Notably, although verbal discourse about mathematical ideas can scaffold learning, the sudden turns in dialogue are difficult to script (Renne, 2004). Such conversations are reliant on deep conceptual understanding, on teachers' and students' ability to effectively articulate mathematical ideas, and on the adaptability of the teacher.

In a similar conversation, another student stated that a prime is "a number that can be divided by itself or one" and gave examples: "1, 3, 5, 7, 9 um …, 11 …". Notably, the number two was omitted, one was included, and the student was unsure whether nine was prime or not. Moreover the word *or* was used in her definition rather than *and*, an example of the need for precision in mathematical language. In mathematics, minor words such as *or* may change the meaning of a statement, an issue described by Zevenbergen (2004). Further questions would have been appropriate here to ascertain whether, the student's difficulties in describing the concept corresponded to an imperfect understanding of the concept.

Together with discussions, the use of representations, four-square model diagrams, and dictionaries have the potential to stimulate construction of understanding of terms such as *prime number*, clarifying that prime numbers exclude one and nine but include two. Descriptions in dictionaries or in peers' work may draw students' attention to the differences between their definitions and examples, and those given in the dictionary or in others' work. For instance, Eather (2011) stated that a prime is a number that has two factors, divisible by only itself and 1. Included are examples of prime numbers: 2, 3, 5, 7, 11... Such descriptions provide a means of identifying and correcting student misconceptions. Notably, the definition of prime should perhaps state that a prime number has exactly two factors, one and itself.

Educators advocate that concrete manipulatives then diagrams are employed to enhance understanding of concepts, prior to abstractions of a concept (e.g., Heath, 2010; Van de Walle, Karp, & Bay-Williams, 2010). Understanding of a term such as *prime number* can be scaffolded with counters. Prime numbers can only be represented by a straight line of counters, and not by a rectangle of counters (see Figure 2 below). The number 7, a prime number, can only be represented in a straight line having factors 1 and 7. On the other hand 12, a composite number, can be represented by a number of arrays and has factors 1, 2, 3, 4, 6, 12. This is an example of the power of a representation to enhance understanding of mathematical concepts.

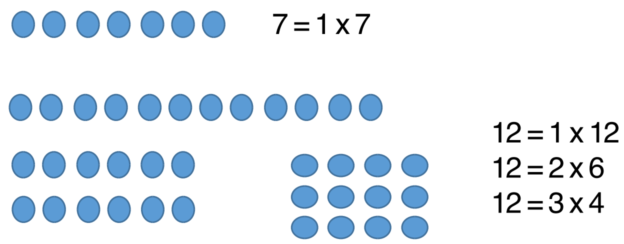

Figure 2. Representation of a prime number 7 and a composite number 12 with arrays of counters.

Alternatively, primes can be represented by starting with a mixture of prime and composite numbers and listing all the factors for each. This reveals the prime numbers, which have exactly two factors, and shows that one, which has only one factor, and nine, which has three factors, are not prime (see Table 2 below).

Table 2
Identifying Prime and Composite Numbers by their Number of Factors

Number	Factors	Prime or Composite
1	1	Neither prime nor composite
2	1, 2	Prime
3	1, 3	Prime
4	1, 2, 4	Composite
5	1, 5	Prime
6	1, 2, 3, 6	Composite
7	1, 7	Prime
8	1, 2, 4, 8	Composite
9	1, 3, 9	Composite

Other strategies that can be used to deepen understanding of mathematical concepts include giving learners opportunities to read, write, and communicate in mathematics in order to aid learners to gradually move towards competently using, with a deep understanding, increasingly formal mathematical language. Unlocking understanding is a way of moving away from student difficulties such as learners misusing or forgetting formulas and procedures; for instance, multiplying the dimensions of a rectangle to calculate the perimeter, locating a median without first ordering the data values, or misinterpreting everyday text that refers to averages (see Brahier, 2009).

When carefully chosen, language strategies such as those discussed provide opportunities to stimulate mathematically oriented debate and discussion and to enhance learners' competencies to describe mathematical concepts concisely and accurately. Relevant activities and dialogue depend on a deep engagement with the subtle and precise meanings of mathematical language. With the use of varied representations, incorporation of written and verbal activities, and scaffolding from other students and teachers, students can be encouraged to collaboratively deepen their understanding of important mathematical language. Promoting such understanding is dependent on exposure to diverse learning experiences and contexts.

Conclusion

Enhancing understanding through a focus on language is crucial to aiding students to build a solid foundation of mathematical ideas. Students require opportunities to engage with written and verbal activities, to aid them to efficiently communicate about mathematical ideas. Such opportunities need to be provided by teachers who have a clear understanding of the literacy demands of mathematics.

Knowledge of strategies that can be used to subtly scaffold students' immature conceptual understanding of mathematical ideas is crucial for teachers, to enable them to scaffold understanding in the subject. Strategies such as those described incorporate verbal and written discourse, with an emphasis on the use of concrete materials, diagrams, descriptions, examples, and symbolic representations. Mathematical handbooks and dictionaries are useful resources in such activities.

Learners gradually develop an understanding of concepts by using and exploring the meanings and multiple meanings of words and symbols in many circumstances, enabling them to move towards more precise use and understanding of vocabulary and abstract concepts (Kotsopoulos, 2007; Zazkis et al., 2009). This aligns with the focus on understanding in the Australian Curriculum (Australian Curriculum, Assessment and Reporting Authority, 2018).

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